

Learning Objectives:

At the end of this topic you will be able to:

- ☑ use ideas of potential and kinetic energy for a space probe in orbit around the Sun;
- ☑ understand why it requires a lot of energy to reach other parts of the Solar System;
- ☑ describe the way in which planetary fly-bys are used to boost a space probe's speed;
- ☑ explain how moving past a planet can boost the speed (and energy) of a space probe;
- ☑ do calculations on speed boost in a sling shot in simple cases.

Kinetic and Potential Energy revision

You'll remember from the Physics 2 course that

- an object which is above the ground has potential energy and that the formula for potential energy is:

$$E = mgh,$$

where m = mass, g = gravitational strength, h = height.

- a moving object has kinetic energy and that this is calculated using the formula

$$E = \frac{1}{2}mv^2,$$

where m = mass, v = speed.

- if an object is thrown through the air its total energy is always the same, as long as air resistance is negligible.

Example: A 5 kg rock is dropped from a height of 100 m. Calculate

- its potential energy when it is dropped,
- its kinetic energy as it hits the ground,
- its speed when it hits the ground.

[Take $g = 10 \text{ N/kg}$]

Answer: (a) Potential energy at start, $E = mgh = 5 \times 10 \times 100 = 5000 \text{ J}$

(b) K.E. at ground level = P.E. at start = 5000 J

(c) To calculate speed:

$$\text{Kinetic energy } E = \frac{1}{2}mv^2, \text{ so } 5000 = \frac{1}{2}5v^2$$

$$\therefore v^2 = \frac{2 \times 5000}{5} = 2000 (\text{m/s})^2$$

$$\therefore \text{speed, } v = \sqrt{2000} = 44.7 \text{ m/s}$$

A couple of questions for you to try.

1. A model aeroplane has a mass of 2.5 kg. It is flying at a height of 200 m with a speed of 20 m/s. Calculate (a) its potential energy, (b) its kinetic energy, (c) its total energy.
2. An archer fires a 200 g arrow vertically upwards with a speed of 50 m/s. Calculate (a) its kinetic energy, (b) the height that it reaches.

In both these questions, ignore air resistance and take gravitational strength to be 10 N/kg.

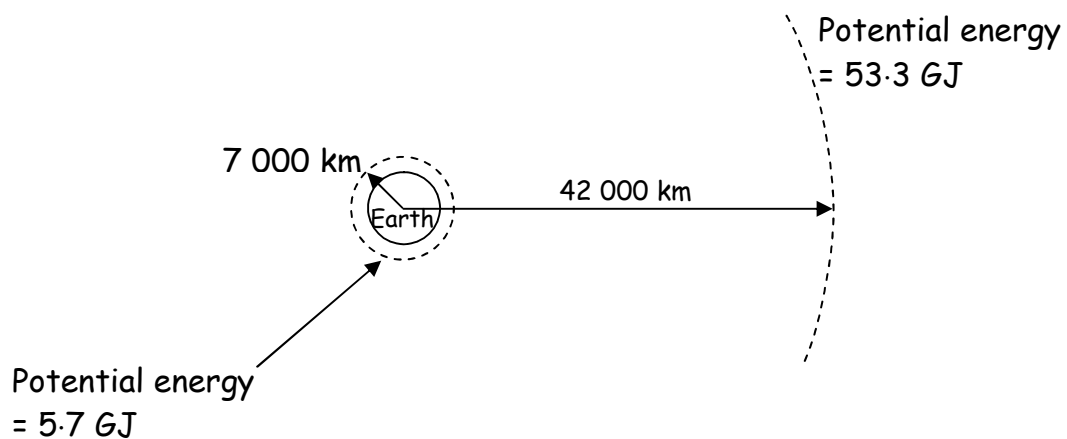
Kinetic and Potential Energy for orbiting objects

The **good** news is that the formula for kinetic energy, $E = \frac{1}{2}mv^2$, is the same - so nothing new here.

The **bad** news is that potential energy is more complicated. Why? The equation $E = mgh$ only works if the gravitational force is constant. When we are quite close to the Earth's surface, this is a very good approximation¹ but we'll be dealing with spacecraft that vary their distance from the Sun by very large amounts - 100s of millions of kilometres - so we won't be able to use $E = mgh$. On the other hand it means that the examiners won't be able to ask you to work out potential energy - so that's quite good news really.

How does potential energy work in space then? Look at the following diagram - not to scale. It shows 2 orbits

- a low Earth orbit about 600 km above the Earth [7000 km from the Earth's centre]
- a Geosynchronous orbit of radius 42 000 km

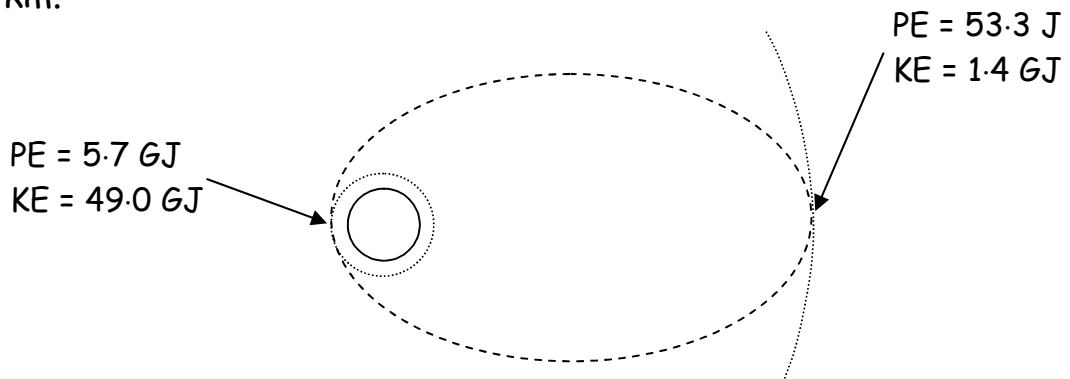


The figures show the potential energy for a 1 tonne [1000 kg] object, e.g. a communications satellite, at that height. Remember $1 \text{ GJ} = 10^9 \text{ J}$. Don't worry about how I worked them out.

¹ Even as high as the orbit of the International Space Station, about 400 km, the value of g has only dropped to 9.5 N/kg.

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Let's think about a satellite orbiting between 7 000 km and 42 000 km.



Look at the figures for Potential Energy and Kinetic Energy.

Example:

Calculate the total energy [i.e. PE + KE] at the lowest point and at the highest point.²

Answers: At lowest point, $E_{\text{Tot}} = 5.7 + 49.0 = \dots\dots\dots \text{GJ}$

At highest point, $E_{\text{Tot}} = 53.3 + 1.4 = \dots\dots\dots \text{GJ}$

What do you notice? Of course they are the same - the Principle of Conservation of Energy works everywhere, even in space. In fact we don't even have to say, "Ignoring air resistance," because there isn't any!

Self-test questions:

3. A 100 kg satellite is orbiting in a circular orbit with a radius of 7 000 km. Its potential energy is 283 MJ and its kinetic energy is 2.86 GJ.
 - (a) Calculate its orbital speed.
 - (b) How much energy does it have to be given to take it up from ground into orbit?
 - (c) Suggest why much more chemical energy has to be used than your answer to (b).
4. A 50 kg meteoroid is in an elliptical orbit around the Sun. Complete the table with information about its energy at different positions:

Position	Distance from Sun (million km)	Potential Energy (GJ)	Kinetic Energy (GJ)	Total Energy (GJ)
Aphelion (see note ²)	250	26.8	19.1	
Perihelion	100		3.1	
At Earth's orbit	150	35.7		

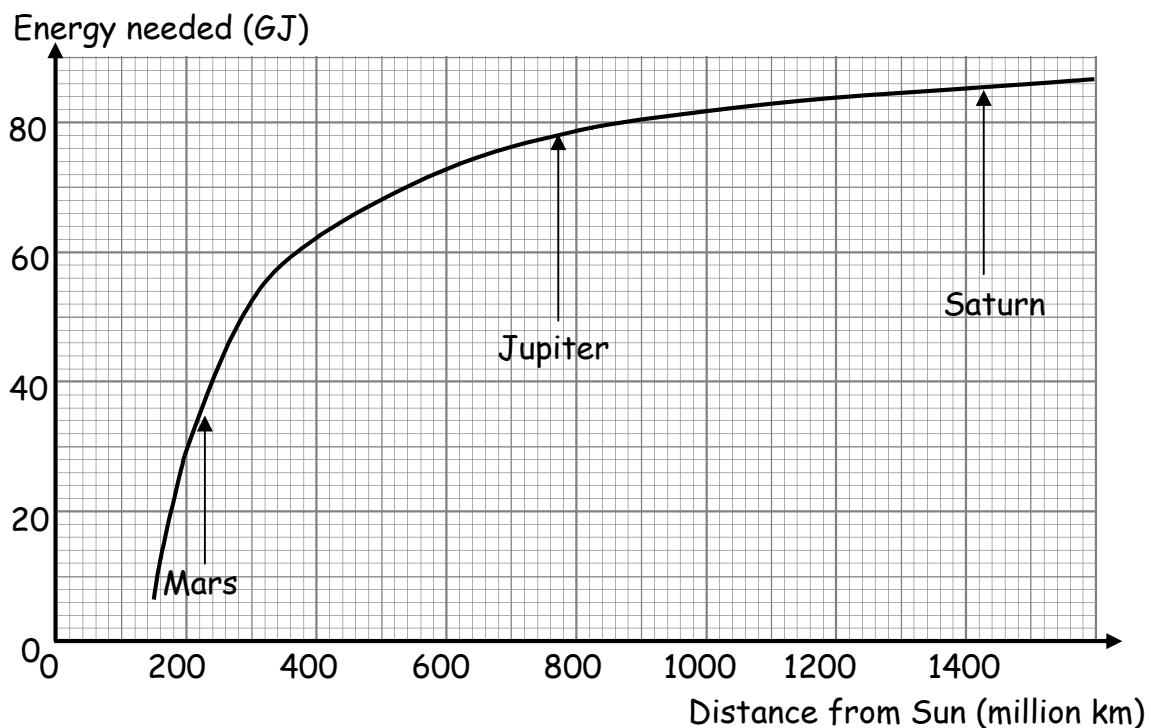
How fast is the meteoroid going when it crosses the Earth's orbit?

² Note for geeks: The closest point to the Earth is called the *perigee* and the furthest point is the *apogee*. For a body in orbit around the Sun, the closest point is the *perihelion* and the furthest point is the *aphelion*. For an object orbiting a star we have *astron* and

The energy gap - or why we need slingshots.

If we want a 100 kg space probe to escape from the Earth's gravitational pull, we need to give it more than 6.3 GJ of energy. To climb outwards in the Solar System against the Sun's gravity, it needs a lot more.

Look at this graph which shows how much energy we need to give to a 100 kg spacecraft on the Earth to allow it to travel to different distances from the Sun.



We can calculate how fast we have to accelerate the spacecraft to allow it to reach one of these planets.

Example: Jupiter:

From the graph, the gain in Potential energy needed = 78 GJ.

So Initial Kinetic energy is at least 78 GJ.

Using $KE = \frac{1}{2}mv^2$, $78 \times 10^9 = \frac{1}{2} \times 100 \times v^2$

So $v^2 = 1.56 \times 10^9$, so $v = 40\,000$ m/s (approx) i.e. 40 km/s

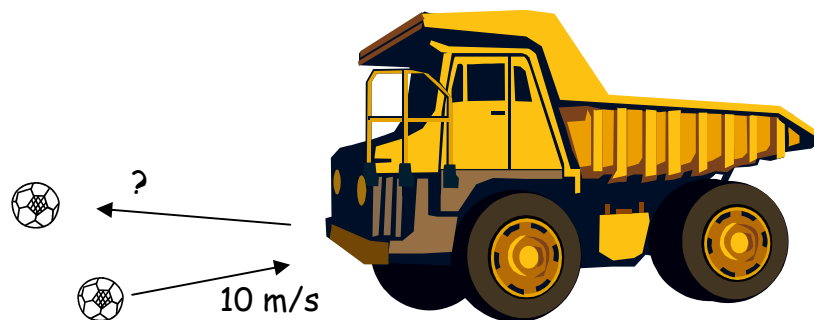
So that's the problem - it takes a lot of energy. The more energy that is needed, the more expensive the mission.

The solution - steal a bit of energy from another planet!

This is not as crazy as it sounds - it is just what space engineers do. They let a spacecraft fly past a planet and in doing so it picks up energy. Do it several times and you pick up enough energy to get to Jupiter. If you're clever enough you can pick up energy from Jupiter to get to Saturn and then onto Uranus..... This is just what the Voyagers did 20 years ago.

So what's going on? Let's start by kicking a football against a dumper truck. Let's say at 10 m/s. [This is only a thought experiment!]

How fast does it bounce off?



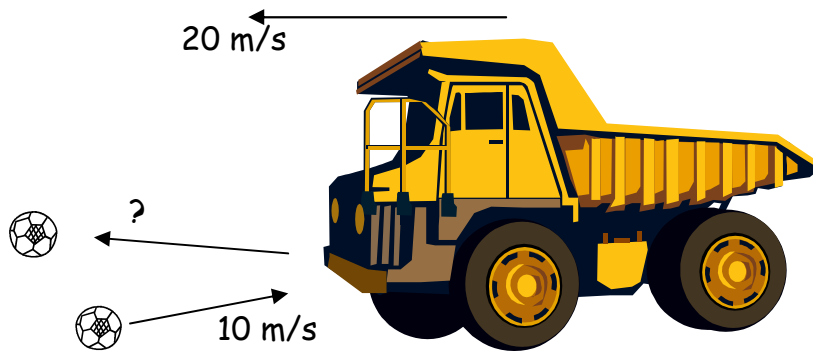
Of course it depends - on how bouncy the ball is, how hard the truck is and how fast the truck is moving.

Let's make it easy: The truck is not moving and the ball is 100% bouncy - i.e. it doesn't lose any energy when it bounces. In this case the answer is obvious - it bounces off at 10 m/s, so it has the same kinetic energy as before. So we haven't won any energy.

[Note - this isn't very realistic, but we'll see when we move on to planets and spacecraft that we can use some answers here].

Now let's step up the difficulty and have the truck moving, let's say at 20 m/s to the left [that's about 45 mph].

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This is suddenly a much more difficult situation, but we can make it easier to understand if we look at the whole thing from the point of view of the driver:

(you might find it easier to think of the driver as not having a speedometer and not having any idea of whether the lorry is moving or not. This is what sometimes happens in space - you might be moving very quickly but all around you is dark and there's no whistling of the wind so you don't know whether or not you're moving)

Question: What does the driver see?

Answer: He sees a ball flying towards him.

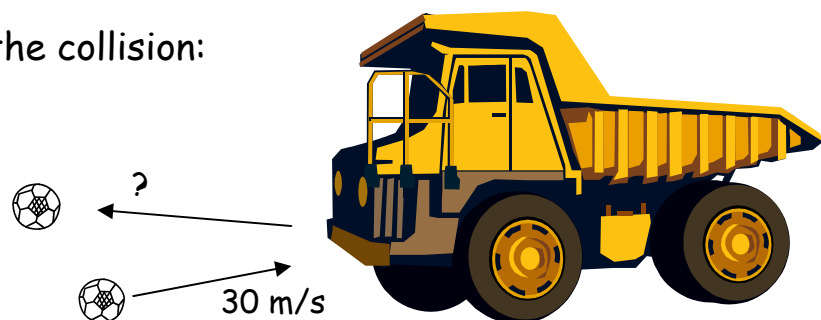
Question: How fast is the ball coming towards him?

Answer: 30 m/s [can you see how we get this figure?]

So, from the point of view of the driver, the ball is whizzing towards the dumper truck at 30 m/s

The driver's view of the collision:

So how fast does the ball bounce off from the driver's point of view?



Answer: 30 m/s. [The driver believes in Conservation of Energy!]

So the ball is bouncing away from the driver at 30 m/s - i.e. it is moving to the left **30 m/s faster than the lorry**, i.e. at 50 m/s! We can now fill in the bouncing off speed in the top diagram.

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The ball is now moving 5 x as fast as before - its kinetic energy is 5^2 as much as before, i.e. 25 x.

Self test questions:

5. Work out what how fast the ball bounces off in the following situations:

		Before collision		Ball speed after collision
		Ball speed →	Truck speed ←	
(a)		5 m/s	10 m/s	
(b)		12 m/s	5 m/s	

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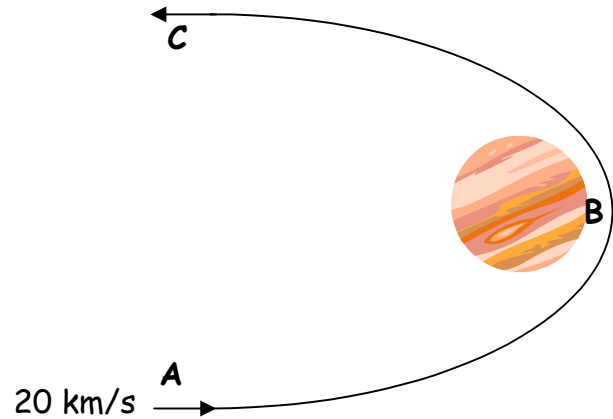
Back to planets and spacecraft: What happens if a spacecraft approaches (but just misses!) a planet.

What happens to its speed as it goes around the planet before heading off back into space?

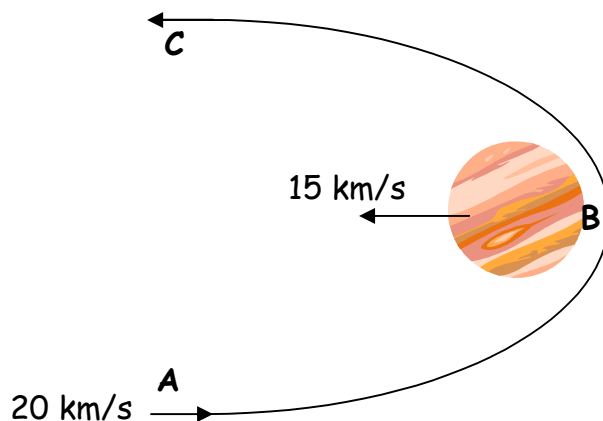
From **A** to **B**, it loses potential energy so picks up kinetic energy.

From **B** to **C**, it gains the same amount of potential energy as it lost before, so its kinetic energy at **C** is the same as at **A**, i.e. 20 km/s.

So the spacecraft escapes from the planet at the same speed it approached - just like the ball and the truck only this time there really is no loss of energy - no air resistance!



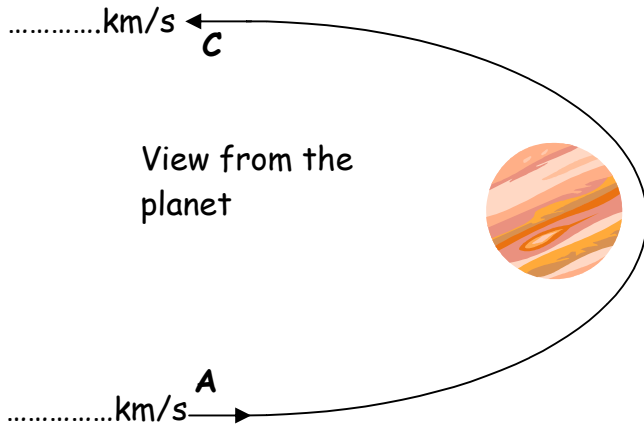
In a real situation the planet will be moving as well in its orbit around the Sun. Suppose the planet is moving at 15 km/s to the left as shown.



How will this affect the spacecraft's speed as it escapes from the planet's gravity?

It's just like the truck. The trick is to see the situation from the point of view of an observer on the planet: [see next page]

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From the point of view of an observer on the planet:

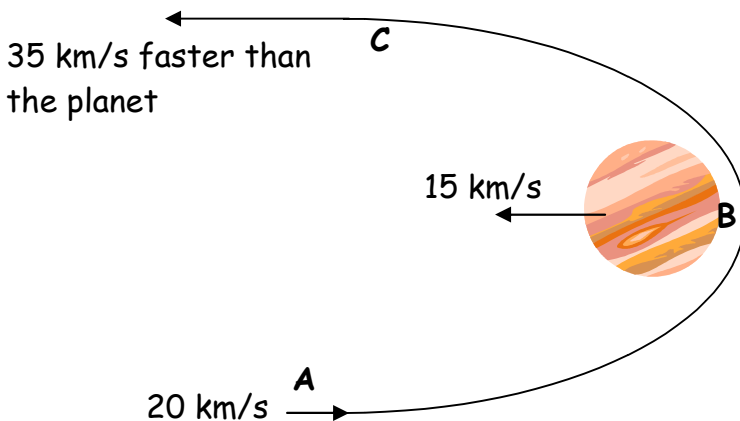
How fast is the spacecraft at **A** approaching the planet?

Answer: 35 km/s

Can you see why?

So, by the time it gets to **C**, the spacecraft is **moving away** from the planet at 35 km/s, as seen from the planet. Fill in the figures.

Now we'll switch back to looking at the whole thing from the Solar System's point of view:



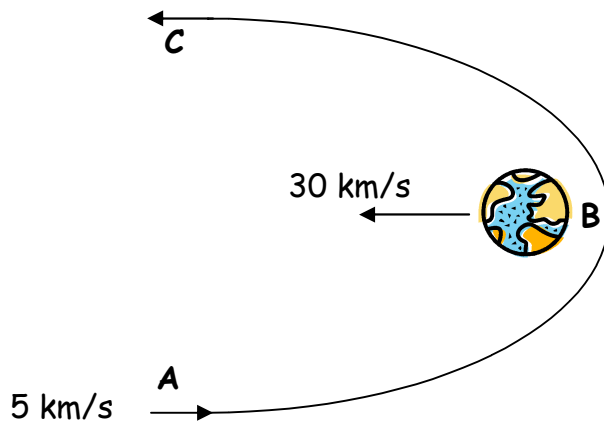
The spacecraft is moving 35 km/s faster than the planet. The planet is moving at 15 km /s, so the total speed of the spacecraft is:.....

.....50 km/s

Now here's one for you to work out:

A recent space shot visited one of the asteroids. It was sent off into orbit round the Sun and its orbit was chosen so that, after a few months, it swung around the Earth again and received a boost. The situation is like that shown in the first diagram on the next page:

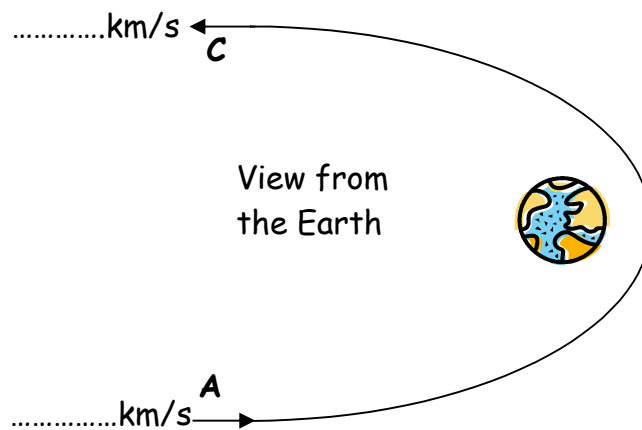
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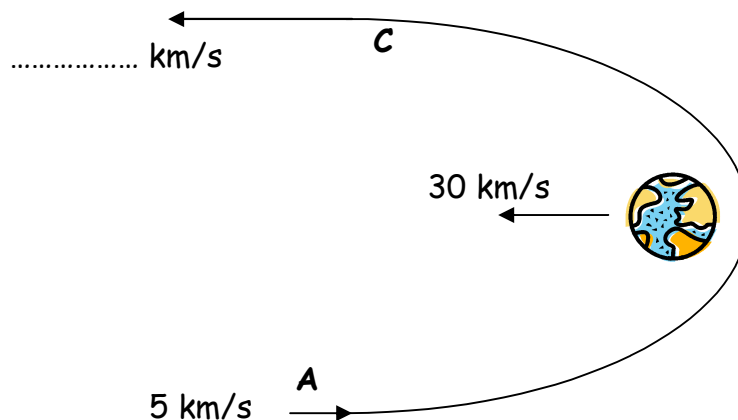
First question:
From the point of view of an observer on the Earth, how fast is the spacecraft approaching the Earth at **A**?

Answer:km/s

On the next diagram, put the speeds of the spacecraft at **A** and **C**, as seen from the Earth.



Last part: you've worked out how much faster the spacecraft is at **C** than the Earth is. Now put in the speed of the spacecraft at **C** from the point of view of the Solar System.



An important question: where does the extra energy come from?

The football on page 7 picked up energy. Where did it come from? Obviously the dumper truck. If the truck is not accelerating we could use the *Principle of Conservation of Momentum* to work out how fast the truck and ball were moving after the collision. The truck has a mass of about 20 000 kg and the ball 1 kg. How much do you think the truck will slow down? Answer: *not a lot* ! But that's where the energy comes from.

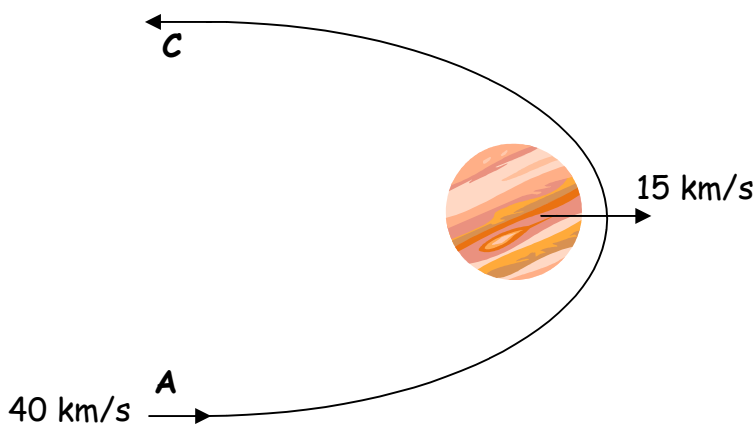
What about the satellite. Where do you think the extra energy comes from?

Answer?

That's right - from the planet. Now the mass of the Earth is 6×10^{24} kg - in other words about 60 000 000 000 000 000 000 000 times the mass of the satellite - so we do nick a bit of the planet's energy, but you'd be able to do it every day for a million years and still have stolen less than 0.000 000 000 001% of its energy. [Don't get hung up on these numbers]

Next important question: can you slow a satellite down?

Answer: Very easily - look at the following situation:



By going through the same process as before, you should be able to show that the spacecraft's speed after the encounter with Jupiter is 10 km/s to the left. Try it.

So its speed and kinetic energy have decreased. Where has the energy gone?

3rd Question: Why would you want to slow a spacecraft down?

Answer: Think about visiting Venus or Mercury. Why would you need to slow down a spacecraft? Discuss!

4th Question: How many more questions are there?

Answer: Only 1 more.

5th Question: What happens if the spacecraft approaches a planet from the side rather than from the front or back?

Answer: Ah! Saving the difficult one until last, eh? This is actually what happens most of the time. The good news is that there are no more sums. NASA has to do them, but we don't.

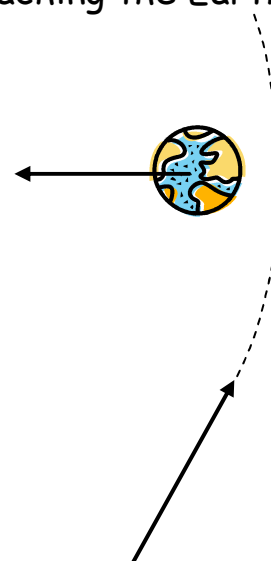
Let's have a look.....think about a satellite approaching the Earth:

This is what the situation looks like from the point of view of the Solar System.

Doing the same as before. How would it look to you or me on the Earth?

Can you see that it would seem to be coming faster and more "horizontal" than in this diagram?

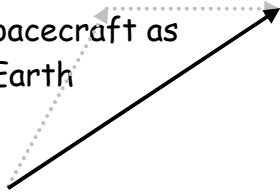
In fact like the diagram on the next page!



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Speed of spacecraft as seen from Earth



Who put those grey dotted arrows there? Ignore them - they are just to make your Physics teacher happy! Has it worked? So what happens after the satellite has gone past? Look at the next diagram below.....

As seen from the Earth, the satellite moves away at the same speed that it approached.

Now remember that the Earth itself is moving towards the left - so the true speed and direction of the departing satellite is:

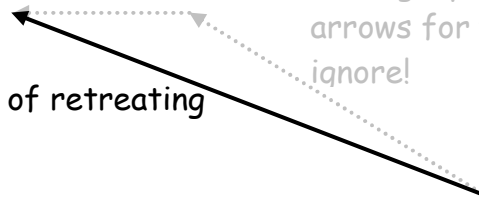
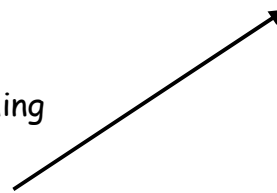
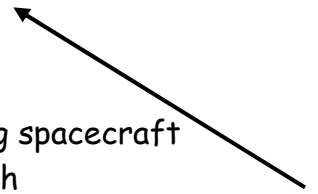
So it has speeded up but if you (or NASA) do the sums, you'll find that it hasn't sped up as much as if it had been coming from straight ahead.

Speed of retreating spacecraft - as seen from Earth

Speed of approaching spacecraft

True speed of retreating spacecraft

More grey dotted arrows for you to ignore!



So what do you need to know in answer to question 5?

Nothing more than the paragraph above - you can use the planet to speed up if you approach from angles to the side - it just gains a bit less energy than a head-on approach.